1 أولى Differential equation

المعادلات التفاضلية

الحصة 1

$y' = 0$ $\qquad$ $y'' + 2y' + 4 = 0$

$y'' + 2xy' = 4$

order — أكبر مشتقة في الدالة

$y''' + 2x(y'')^7 + 4 = 0$

3rd order , 1st deg

Degree — الأس لأكبر مشتقة

$y''' + 2x(y'''')^7 = x(y''')^8$

4th order $\qquad$ 7th degree

Prove that

$y = Ae^{x} + Be^{2x}$ Solution

$y'' - 3y' + 2y = 0$

$y' = Ae^{x} + 2Be^{2x}$

$y'' = Ae^{x} + 4Be^{2x}$

$y'' - 3y' + 2y$

$= (\cancel{Ae^{x}} + \cancel{4Be^{2x}}) \cancel{- 3Ae^{x}} - \cancel{6Be^{2x}}$

$+ \cancel{2Ae^{x}} + \cancel{2Be^{2x}} = 0$

c

Solution of D.e

حل المعادلة التفاضلية

D.e هو المعادلة التي تحقق

Prove that $y = A\cos x + \beta \sin x$

Solution $y'' + y = 0$

$$y' = -A\sin x + \beta\cos x$$

$$y'' = -A\cos x - \beta\sin x$$

$$y'' = -[A\cos x + \beta\sin x]$$

$$y'' = -y$$

$$y'' + y = 0$$

Formation of O.D.E by eleminting Constant

تكوين معادلات التفاضلية بحذف الثوابت

- نفاضل بعدد مرات الثوابت

$$y = Ax^2$$

$$y' = 2Ax$$

$$\therefore y' = 2\frac{xy}{x^2}$$

$$y' = \frac{2y}{x}$$

$$y = Ae^{x} + Be^{-x}$$

$$y' = Ae^{x} - Be^{-x}$$

$$y'' = \underline{Ae^{x} + Be^{-x}}$$

$$\therefore y'' = y$$

$$y'' - y = 0$$

Find o.D.e if

$$y = Ax + \frac{B}{x}$$

$$y' = A - \frac{B}{x^2}$$

$$y'' = 0 + \frac{2B}{x^3}$$

$$B = \frac{y'' x^3}{2}$$

$$\therefore y' = A - \frac{x y''}{2}$$

$$A = y' + \frac{x y''}{2}$$

$$\therefore y = x y' + \frac{x^2 y''}{2} + \frac{y'' x^2}{2}$$

$$y = x y' + x^2 y'' \qquad 0$$

obtain o.D.e if

$$y = Ax^3 + Bx^2 + C$$ is

Solution of D.e

$$y' = 3Ax^2 + 2Bx$$

$$y'' = 6Ax + 2B$$

$$y''' = 6A$$

$$\therefore y'' = xy''' + 2B$$

$$\therefore y' = 3\frac{y'''}{6}x^2 + y'' - xy'''$$

Solution of 1^{st} order 1^{st} degree

$$y' = f(x, y)$$

$\boxed{1}$ sepration of varible
فصل المتغيرات

Solve D.e

خاص بـ y

$$\frac{dy}{dx} = \frac{x}{y}$$

خاص بـ x

$$\int y \, dy = \int x \, dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$y(0) = 2$$

at $x=0 \quad y=2$

$$\frac{4}{2} = c$$

$$\frac{y^2}{\cancel{2}} = \frac{x^2}{\cancel{2}} + \cancel{\frac{4}{2}}$$

Solve

$$\frac{dy}{dx} = e^{x-y} = e^{x} \cdot e^{-y}$$

$$\frac{dy}{dx} = \frac{e^x}{e^y}$$

$$\int e^y dy = \int e^x dx$$

$$e^y = e^x + c$$

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$$\frac{dy}{dx} = xy + x + y + 1$$

$$\frac{dy}{dx} = x(y+1) + (y+1)$$

$$\frac{dy}{dx} = (y+1)(x+1)$$

$$\int \frac{dy}{y+1} = \int (x+1)\,dx$$

$$\ln(y+1) = \frac{(x+1)^2}{2} + c$$

$$\sin x \cosh y \, dx + \sinh y \cos x$$

$$=$$

$$\div \cosh y \cdot \cos x$$

$$\int \frac{\sin x}{\cos x}\,dx + \int \frac{\sinh y}{\cosh y}\,d$$

$$\frac{dy}{dx} = xy + x + y + 1$$

$$\frac{dy}{dx} = x(y+1) + (y+1)$$

$$\frac{dy}{dx} = (y+1)(x+1)$$

$$\int \frac{dy}{y+1} = \int (x+1)\,dx$$

$$\ln(y+1) = \frac{(x+1)^2}{2} + c$$

$$\sin x \cosh y \, dx + \sinh y \cos x \, dy = 0$$

$$\div \cosh y \cdot \cos x$$

$$\int \frac{\sin x}{\cos x}\,dx + \int \frac{\sinh y}{\cosh y}\,dy = 0$$

$$-\ln \cos x + \ln \cosh y = \overset{\ln c}{\cancel{c}}$$

$$\ln \frac{\cosh y}{\cos x} = \ln c$$

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(2) Can be sepration

مع تحويل لـ فصل متغيرات

$$\frac{dy}{dx} = f(ax+by+c)$$

$$\text{let } ax+by+c = u$$ نفرض

$$a + b y' = \frac{du}{dx}$$ نفاضل

<u>Ex</u> $\frac{dy}{dx} = (x+y+3)^2$ (1)

$$x + y + 3 = u$$

$$1 + \frac{dy}{dx} = \frac{du}{dx}$$

$$\therefore \frac{dy}{dx} = -1 + \frac{du}{dx}$$

بالتعويض في ①

$$-1 + \frac{du}{dx} = u^2$$

$$\frac{du}{dx} = u^2 + 1$$

$$\int \frac{du}{u^2+1} = \int dx$$

$$\tan^{-1} u = x + C$$

$$\tan^{-1}(x+y+3) = x + C$$

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$$\frac{dy}{dx} = \tan^2(x+y+1)$$

let $x+y+1 = u$

$$1 + \frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{du}{dx} - 1 = \tan^2 u$$

$$\frac{du}{dx} = 1 + \tan^2 u$$

$$\frac{du}{dx} = \sec^2 u$$

$$\int \frac{du}{\sec^2 u} = \int dx$$

$$\int \cos^2 u \, du = x + c$$

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$$\frac{1}{2}\int (1+\cos 2u)\,du = x + c$$

$$\frac{1}{2}\left[u + \frac{\sin 2u}{2}\right] = x + c$$

$$\frac{1}{2}\left[(x+y+1) + \frac{1}{2}\sin(2x+2y+2)\right. = x + c$$

<u>Solve De</u>

$$\frac{dy}{dx} = \cos(x+y) + \cos(x-y)$$

$$\frac{dy}{dx} = 2\cos x \cos y$$

$$\int \frac{dy}{\cos y} = \int 2\cos x\,dx$$

$$\int \sec y\,dy = 2\sin x + c$$

$$\ln|\sec y + \tan y| = \text{〃}$$